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Backstepping dynamical sliding mode control method for the path following of the underactuated surface vessel

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Abstract

A method of backstepping adaptive dynamical sliding mode control is presented for the path following control system of the underactuated surface vessel. The system consists of the nonlinear ship response model and the Serret-Frenet error dynamics equations. The control system takes account of the modeling errors and external disturbances. It transforms the original underactuated system into a nonlinear system via simplified analysis. An adaptive dynamical sliding mode controller is proposed based on backstepping method and dynamical sliding mode control theory. By means of Lyapunov function, it is proven that the proposed controller can render the path following control system globally asymptotically stable. Simulation results verify that the controller is robust and adaptive to the systemic variations or disturbances

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Keywords: underactuated surface vessel(USV); path following; dynamical sliding mode control; backstepping method; adaptive

1. Introduction

This paper addresses the problem of path following for underactuated surface vessel (USV). The challenging problem is how to control the three freedom motions by using only two independent inputs [1]. Path following control has received relatively less attention than trajectory tracking problem. The USV path following problem has been addressed with two different methods: one is to treat it as a tracking control problem [2, 3, 4], and the other is to simplify the tracking control problem into a regulation control problem by adopting proper path following error dynamics [5, 6, 7]. For the latter approach, the Serret-Frenet frame is often adopted to derive the error dynamics. In [8], a fourth order ship model subject to a constant known direction ocean-current disturbance in the Serret-Frenet frame was used to develop a control strategy to track both the straight line and the circumference. The authors in [9] have presented a controller based on a transformation of the ship kinematics to the Serret-Frenet frame on the path, where an acceleration and linearization of ship dynamics were used. Do and Pan [10] proposed

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an output feedback path following strategy, where ultimate convergence is proven for an underactuated ship under environmental disturbances. However, the proposed method in [10] requires a state transformation, which becomes singular at some configuration. Such a restriction is obstructing, especially from a theoretical point-of-view, and effectively excludes the derivation of any global path following results. In [11], a simplified vessel model was used to develop a path following control system, the authors has presented a controller based on backstepping technique and Lyapunov direct method. However, the simplified vessel model ignores the influence of nonlinear roll motion.

We transform the path following problem of the underactuated system into stabilization problem of the nonlinear system by simplified analysis of the USV system. Based on backstepping method and dynamical sliding mode control theory, a backstepping adaptive dynamical sliding mode controller is proposed. We demonstrate that the original system is globally asymptotically stabilized to the desired configuration with the controller. The advantage of the controller is that control system is strongly robust and adaptive to the modeling errors, systemic variations and disturbances. The effectiveness of the proposed method is illustrated and validated by simulation results on a model vessel.

2. System description and analysis

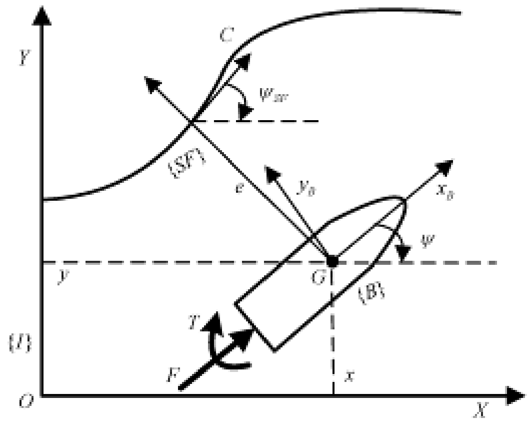

Fig. 1. Path following model of the vessel

The three degree of freedom planar model of the USV shown in Fig.1 is considered in this work. The kinematics and dynamics models of the USV are described by the following ordinary differential equations [12]:

$$\begin{cases} \dot{u} = (m_{22}\upsilon r - d_{11}u + F_u)/m_{11} \\ \dot{\upsilon} = -(m_{11}ur + d_{22}\upsilon)/m_{22} \\ \dot{r} = ((m_{11} - m_{22})u\upsilon - d_{33}r + T_r)/m_{33} \\ \dot{x} = u\cos\psi - \upsilon\sin\psi \\ \dot{y} = u\sin\psi + \upsilon\cos\psi \\ \dot{\psi} = r \end{cases} \quad (1)$$

where x, y denotes the coordinates of the USV in the earth-fixed frame, and ψ is the heading angle, and u, υ, r denote the velocity in surge, sway, and yaw respectively, the surge force F_u and the yaw torque T_r are considered as the control inputs. Parameters m_{ii} and d_{ii} are assumed to be positive constants and are given by the vessel inertia and damping matrices. Clearly, the USV is underactuated because the sway force is missing in the υ-equation (1).

The origin of the Serret-Frenet frame {SF} is located at the closest point on the curve C from the origin of frame {B}. The error dynamics based on the Serret-Frenet equations are given by [9]

$$\begin{cases} \dot{\bar{\psi}} = \dot{\psi} - \dot{\psi}_{SF} = \kappa(u\sin\bar{\psi} - \upsilon\cos\bar{\psi})/(1-e\kappa) + r \\ \dot{e} = u\sin\bar{\psi} + \upsilon\cos\bar{\psi} \end{cases} \tag{2}$$

where e defined as the distance between the origins of {SF} and {B}, and $\bar{\psi} = \psi - \psi_{SF}$ are referred to as the cross-track error. ψ_{SF} is the path tangential direction, and κ is the curvature of the given path.

For most path following problems for USV in open sea, the path is a straight line or piecewise straight lines with the curvature $\kappa = 0$. Therefore, the heading error dynamics can often be simplified as [11]

$$\dot{\bar{\psi}} = r \tag{3}$$

Remark 1. Notice that the rudder angle is the control input, while the yaw torque is used as the input in (1). However, the rudder angle is a real actuator variable, but the yaw torque is not. In general, one order nonlinear ship roll response model is used to design the ship steering system [13].

The roll response model takes account of modeling errors, external disturbances, and rudder actuator dynamics. The USV path following mathematical model can be described as follows

$$\begin{cases} \dot{e} = u\sin\bar{\psi} + \upsilon\cos\bar{\psi} \\ \dot{\bar{\psi}} = r \\ \dot{u} = (m_{22}\upsilon r - d_{11}u + F_u)/m_{11} \\ \dot{\upsilon} = -(m_{11}ur + d_{22}\upsilon)/m_{22} \\ \dot{r} = (-r - \alpha r^3 + K\delta)/T + F \\ \dot{\delta} = (-\delta + K_E\delta_E)/T_E \end{cases} \tag{4}$$

where T, K, α are maneuverability parameters, δ is rudder angel, and δ_E is control rudder angel. T_E, K_E are rudder actuator constants. F is uncertainty summation of the modeling errors Δ and external disturbances ω, namely, $F = \Delta(\psi, \dot{\psi}) + \omega$, we suppose $|F| \le \bar{F}$, and $\dot{F} = 0$.

Remark 2. To simplify the analysis, we assume u is positive constant. Normally, an independent control system is used to maintain the vessel's surge speed. The constant u assumption is adopted by many pursuers. In vessel maneuvering, the υ is relatively small compared to other motion variables. Therefore, we assume that $\upsilon = 0$ [9].

According to Remark 1 and 2, the USV path following control model (4) has been simplified into

$$\begin{cases} \dot{e} = u\sin\bar{\psi} \\ \dot{\bar{\psi}} = r \\ \dot{r} = (-r - \alpha r^3 + K\delta)/T + F \\ \dot{\delta} = (-\delta + K_E\delta_E)/T_E \end{cases} \tag{5}$$

Remark 3. Obviously, the path following problem of the USV is transformed into the stabilization control problem of the nonlinear system (5). Hence, we thereafter need to design the control law δ_E that stabilizes the system (5).

3. Control system design

3.1. Backstepping adaptive dynamical sliding mode controller design

We design the control law based on backstepping technique and dynamical sliding mode control method [14]. Consider the subsystem of the system (5)

$$\dot{e} = u\sin\bar{\psi} \tag{6}$$

$\bar{\psi}$ is virtual control input, in order to eliminate the nonlinear term $\sin\bar{\psi}$, we design the control law $\bar{\psi}$ as follows

$$\bar{\psi} = f(e) = \arctan(-ke) \tag{7}$$

where k is positive constant. Substituting (7) into (6), the (6) becomes

$$\dot{e} = u\sin\bar{\psi} = u\sin[\arctan(-ke)] = -uke/\sqrt{1+(ke)^2} \tag{8}$$

Define Lyapunov candidate function as

$$V_1 = e^2/2 \tag{9}$$

Differentiating V_1 with respect to time yields

$$\dot{V}_1 = e\dot{e} = -uke^2/\sqrt{1+(ke)^2} \tag{10}$$

The system (6) is globally asymptotically stabilized with the control law (7). Let error variable

$$z_2 = \bar{\psi} - f(e) = \bar{\psi} - \arctan(-ke) \tag{11}$$

Differentiating z_2 with respect to time, we obtain

$$\begin{cases} \dot{e} = -uke/\sqrt{1+(ke)^2} \\ \dot{z}_2 = r - uk^2 e/[1+(ke)^2]^{3/2} \end{cases} \tag{12}$$

Define Lyapunov candidate function as

$$V_2 = V_1 + z_2^2/2 \tag{13}$$

We choose the feedback control law r as follows

$$r = f(e,\bar{\psi}) = \arctan(-ke) + uk^2 e/[1+(ke)^2]^{3/2} - \bar{\psi} \tag{14}$$

Differentiating V_2 with respect to time, substituting (14) into $\dot{V}_2$ becomes

$$\dot{V}_2 = \dot{V}_1 + [\bar{\psi} - \arctan(-ke)]\cdot\{r - uk^2 e/[1+(ke)^2]^{3/2}\} = -uke^2/\sqrt{1+(ke)^2} - z_2^2 \tag{15}$$

Obviously, the system (12) is globally asymptotically stable. Let error variable

$$z_3 = r - f(e,\bar{\psi}) = \bar{\psi} + r + Q_1 \tag{16}$$

where $Q_1 = -\arctan(-ke) - uk^2 e/[1+(ke)^2]^{3/2}$, we have

$$\begin{cases} \dot{e} = -uke/\sqrt{1+(ke)^2} \\ \dot{z}_2 = \bar{\psi} - uk^2 e/[1+(ke)^2]^{3/2} \\ \dot{z}_3 = r + \dot{r} + Q_2 \end{cases} \tag{15}$$

where $Q_2 = -uk^2 e/[1+(ke)^2]^{3/2} + u^2 k^3 e/[1+(ke)^2]^2 - 3u^2 k^5 e^2/[1+(ke)^2]^3$.

Define Lyapunov candidate function as

$$V_3 = V_2 + z_3^2/2 \tag{16}$$

Differentiating V_3 with respect to time yields

$$\dot{V}_3 = \dot{V}_2 + z_3(\dot{\bar{\psi}} + \dot{r} + Q_2) \tag{17}$$

We design the feedback control law $\dot{r}$ as follows

$$\dot{r} = f(e,\overline{\psi},r) = -2r - \overline{\psi} - Q_1 - Q_2 \tag{18}$$

Substituting control law (18) into (17), the (17) becomes

$$\dot{V}_3 = -uke^2 / \sqrt{1+(ke)^2} - z_2^2 - z_3^2 \tag{19}$$

The system (18) is globally asymptotically stabilized with the control law (15). Let error variable

$$z_4 = \dot{r} - f(e,\overline{\psi},r) = \overline{\psi} + 2r + \dot{r} + Q_1 + Q_2 \tag{20}$$

The system (5) is eventually transformed to

$$\begin{cases} \dot{e} = -uke / \sqrt{1+(ke)^2} \\ \dot{z}_2 = \overline{\psi} - uk^2 e / [1+(ke)^2]^{3/2} \\ \dot{z}_3 = r + \dot{r} + Q_2 \\ \dot{z}_4 = Q_3 + b_1\delta_E + F_1 \end{cases} \tag{21}$$

where $Q_3 = P_1 + P_2$, we define $a_1 = -1/T, a_2 = -\alpha / T, a_3 = K/T, a_4 = -1/T_E, b = K_E / T_E$, $b_1 = a_3 b, F_1 = -a_4 F, P_1 = (3a_2 r^2 \dot{r} - a_1 a_4 r - a_2 a_4 r^3) + (a_1 + a_4)\dot{r} + r + 2\dot{r}, P_2 = \dot{Q}_1 + \dot{Q}_2$.

Define Lyapunov candidate function as

$$V_4 = V_3 + z_4^2 / 2 + (F_1 - \hat{F}_1)^2 / 2 \tag{22}$$

where $\hat{F}_1$ is estimate value of the unknown uncertain term F_1.

We choose one order dynamical sliding mode switch function as follows, where c_1 is positive constant

$$S = c_1 z_4 + Q_3 + b_1\delta_E + \hat{F}_1 \tag{23}$$

Collecting the system (21) and (23), we have

$$\dot{z}_4 = S - c_1 z_4 + (F_1 - \hat{F}_1) \tag{24}$$

Differentiating V_4 with respect to time, substituting (14) into $\dot{V}_4$, we obtain

$$\dot{V}_4 = \dot{V}_3 + z_4\dot{z}_4 - \dot{\hat{F}}_1(F_1 - \hat{F}_1) = \dot{V}_3 + z_4 S - c_1 z_4^2 + (z_4 - \dot{\hat{F}}_1)(F_1 - \hat{F}_1) \tag{25}$$

Differentiating (23) with respect to time, if we select $v = b_1\dot{\delta}_E$, then $\dot{S}$ becomes

$$\dot{S} = c_1\dot{z}_4 + \dot{Q}_3 + b_1\dot{\delta}_E + \dot{\hat{F}}_1 = v + c_1\dot{z}_4 + \dot{Q}_3 + \dot{\hat{F}}_1 \tag{26}$$

Substituting the (21) into (26), we have

$$\dot{S} = v + c_1(Q_3 + b_1\delta_E + F_1) + \dot{Q}_3 + \dot{\hat{F}}_1 \tag{27}$$

Define Lyapunov candidate function as

$$V_5 = V_4 + S^2 / 2 \tag{28}$$

Differentiating V_5 with respect to time yields

$$\dot{V}_5 = \dot{V}_3 + z_4 S - c_1 z_4^2 + (z_4 - \dot{\hat{F}}_1)(F_1 - \hat{F}_1) + S[v + c_1(Q_3 + b_1\delta_E + F_1) + \dot{Q}_3 + \dot{\hat{F}}_1] \tag{29}$$

If we choose dynamical sliding mode control law v as

$$v = -c_1(Q_3 + b_1 u + \hat{F}_1) - z_4 - \dot{\hat{F}}_1 - \dot{Q}_3 - k_s\,\mathrm{sgn}(S) - w_s S \tag{30}$$

where k_s, w_s are positive constants. Substituting the (30) into (29), we obtain

$$\dot{V}_5 = \dot{V}_3 - c_1 z_4^2 - w_s S^2 + (F_1 - \hat{F}_1)(z_4 + c_1 S - \dot{\hat{F}}_1) - k_s |S| \tag{31}$$

We design the adaptive law of the uncertain term F_1 as

$$\dot{\hat{F}}_1 = z_4 + c_1 S \tag{32}$$

Substituting the (32) into (31), the (31) becomes

$$\dot{V}_5 = -uke^2 / \sqrt{1+(ke)^2} - z_2^2 - z_3^2 - c_1 z_4^2 - w_s S^2 - k_s |S| \le 0 \tag{33}$$

By selecting k, c_1, k_s, w_s are positive constants, $\dot{V}_5$ satisfies the $\dot{V}_5 \le 0$. Therefore, the system (21) is globally asymptotically stabilized with the control law (30) and (32). The system (5) is also globally asymptotically stable.

3.2. Backstepping controller design

In this section, we design the path following controller via backstepping method, we suppose uncertain term $F = 0$. Define Lyapunov candidate function as

$$V_6 = V_3 + z_4^2 / 2 \tag{34}$$

Differentiating V_6 with respect to time, substituting (21) into $\dot{V}_6$, we obtain

$$\dot{V}_6 = \dot{V}_3 + z_4 \dot{z}_4 = \dot{V}_3 + z_4 (Q_3 + b_1 \delta_E) \tag{35}$$

We design the feedback control law as follows

$$\delta_E = b_1^{-1}(-Q_3 - k_2 z_4) \tag{36}$$

where k_2 is positive constant. Substituting the (36) into (35), the (35) becomes

$$\dot{V}_6 = \dot{V}_3 - k_2 z_4^2 = -uke^2 / \sqrt{1+(ke)^2} - z_2^2 - z_3^2 - k_2 z_4^2 \le 0 \tag{37}$$

Note that the state outputs e, z_2, z_3, z_4 of the system (21) decay exponentially to zero with the control law (36). Therefore, the original system (5) is globally exponentially stable.

4. Simulation results and analysis

In this section, we carried out some simulations to validate our proposed method for the USV. We used the following vessel model parameters

$$m_{11} = 200\text{kg}, m_{22} = 250\text{kg}, m_{33} = 80\text{kg}\cdot\text{m}^2, d_{11} = 70\text{kg/s}, d_{22} = 100\text{kg/s}, d_{33} = 50\text{kg}\cdot\text{m}^2/\text{s},$$
$$K = 1, T = 2, \alpha = 0.5, K_E = 1, T_E = 1.5.$$

The initial conditions are $x_0 = 0, y_0 = 0, \psi_0 = 0, u_0 = 2\text{m/s}, \upsilon_0 = 0, r_0 = 0$. The rudder mechanical saturation limit ($-30^\circ \le \delta \le +30^\circ$) is incorporated. In the following backstepping adaptive dynamical sliding mode controller referred to as law 1 and backstepping controller referred to as law 2. We choose the parameters of the law 1 as $c_1 = 2.5, k = 0.1, k_s = 0.005, w_s = 0.1$, and the parameters of the law 2 as $k = 0.2, k_2 = 0.5$.

Firstly, the control law 1 is implemented with the 2 degree simplified model (5) (referred to as 2D), and 3 degree non-simplified model (4) (referred to as 3D). Simulations results are shown in Fig. 2. It is shown in Fig. 2 that the law 1 can fleetly track the desired path under different models, the path following error is uniform attenuation, and the motion path is smooth and non-oscillation. However, motion path has slight overshoot under the 3D model. This illustrate that law 1 has good adaptability and robustness. Fig. 2 shows that the variations of the speed u, v are very small under the 3D model. The above analysis verifies that the system simplification dispose is feasible. Fig. 2 shows that rudder output is not a chattering phenomenon. Hence, the proposed methods effectively reduce the chattering problem.

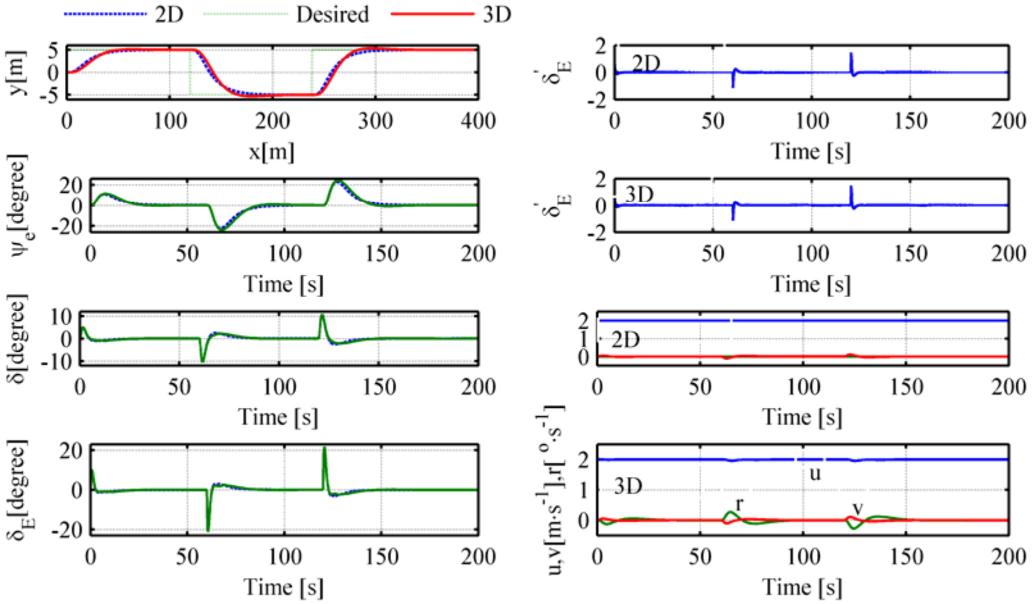

Fig.2. System state response curve under different model

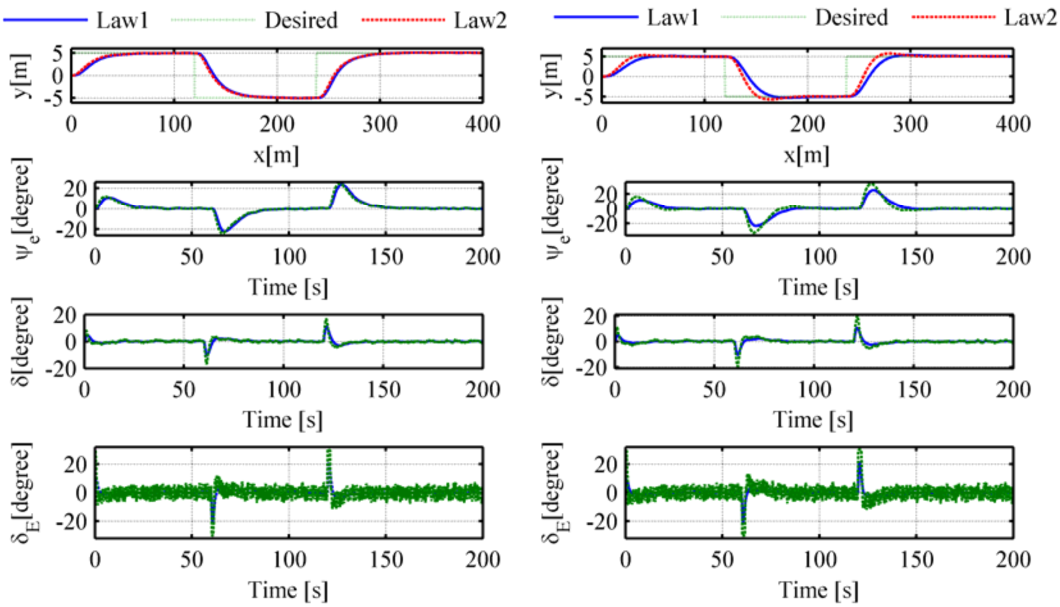

Fig.3. (a) System state response curve under different controller (2D model); (b) System state response curve under different controller (3D model)

In the following simulation, we assume uncertainty input: modeling error is $\Delta = 2\sin(2\pi t)(^{\circ}\cdot s^{-2})$, disturbance force is $\omega = \pm 2(^{\circ}\cdot s^{-2})$. The simulation comparison results of the two control law under different models are shown in Fig. 3.

Fig. 3 (a) shows that the path following is achieved for the law 1 and law 2. Fig. 3 also shows that the rudder output of the law 1 is very smooth. Therefore, law 1 has a strong ability to barrage jamming. Fig. 3 shows that the USV path still can converge to the desired path under different motion model. Comparing law 1 with the law 2, the law 1 has faster convergence, smaller overshoot. The rudder output is also relatively smooth. Therefore, law 1 still has good control performance. Simulation results show that the proposed controller is adaptive and robust to system model perturbation and external interference impact.

5. Conclusions

This paper addressed the path tracking problem of the USV under the influence of modeling errors and unknown external disturbance. Based on certain assumptions, the original underactuated system can be reduced to a non- underactuated nonlinear system. We proposed a backstepping adaptive dynamic sliding mode controller based on backstepping and dynamic sliding mode technique. We proved that the origin system is globally asymptotically stabilized with the controller. Simulation results also illustrated the effectiveness of the proposed control method.

Acknowledgements

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